Lecture 3

Friday, September 3, 2021 11:36 PM

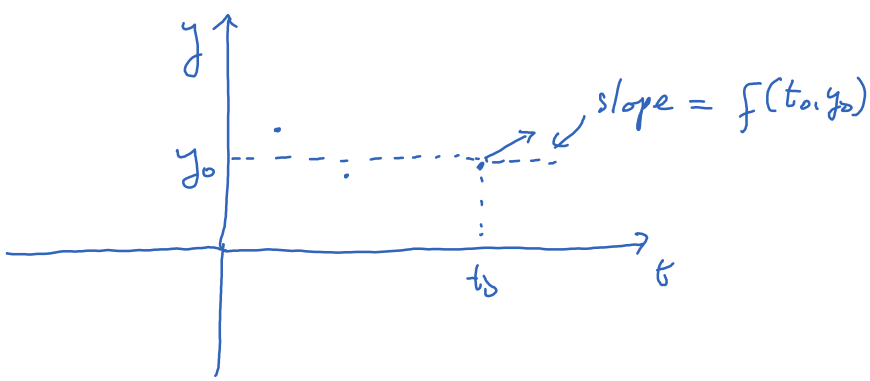
* Prayer
* Spiritual thought: continue with the pattern....

- Population model: $P' = cP$ $P(t)$: # of individual
- Compound interest: $y' = ry$ $y(t)$: account balance at time t
- Heat equation: $u_t = c u_{xx}$ $u(x,t)$: temperature
- Falling object: $y' = g - cy$ $y(t)$: velocity

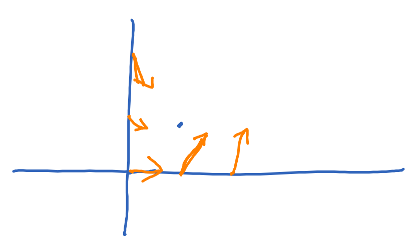
.....

ODE of the form $y' = f(t,y)$.

Geometric representation of the problem:



Ex $y' = \underbrace{2t - y}_{f(t,y)}$



t	y	f(t,y)
0	0	0
1	0	2
0	1	-1
2	0	4
0	2	-2

Use Mathematica to draw the direction field.

VectorPlot[{ 1, f(t, y) }, {t, -1, 1}, {y, -1, 1}]

StreamPlot[......]

Example:

$$y' = 2t - y \,, \quad y(0) = 1$$

Asymptotic behavior of y?

* How to solve for the equation $y' = 2t - y$?

Take a simpler equation : $y' = 2 - y$

$$\frac{dy}{dt} = 2 - y \rightsquigarrow \frac{dy}{2-y} = dt \rightsquigarrow \cdots$$

Linear ODE of first order

$$a(t)\, y' + b(t)\, y + c(t) = 0$$

$$\xrightarrow[\text{form}]{\text{standard}} \quad y' + p(t)\, y = g(t)$$

Integrating factor : $\underbrace{\mu(t)\big(y' + p(t)\, y\big)}_{} = \mu(t)\, g(t)$

want this to be
a derivative of some function

want $\mu(t)\big(y' + p(t)\, y\big) = (\mu y)'$ $\rightsquigarrow$ want $\mu p = \mu' \rightsquigarrow p = \frac{\mu'}{\mu}$

$\rightsquigarrow \int \frac{\mu'}{\mu} dt = \int p\, dt$

$\rightsquigarrow \ln \mu = \int p\, dt$

$\rightsquigarrow \boxed{\mu(t) = e^{\int p\, dt}}$ This is the integrating factor we want.

Ex: $ty' + 2y = t^2$